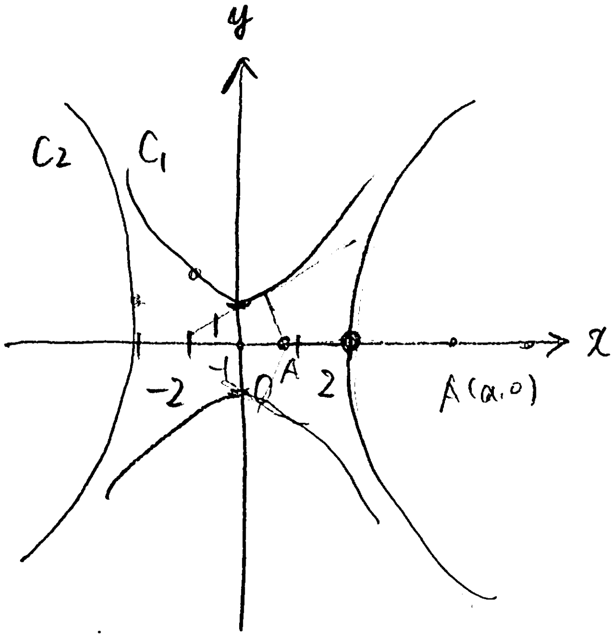
$\boxed{4}$ $a>0$ $A(a,0)$



(1) 点 $P(s,t)$ とする

$$s^2-4t^2=-4$$

$$AP=\sqrt{(s-a)^2+t^2}$$

$$=\sqrt{s^2-2as+a^2+\frac{s^2}{4}+1}$$

$$=\sqrt{\frac{5}{4}s^2-2as+a^2+1}$$

$$=\sqrt{\frac{5}{4}\left(s^2-\frac{8}{5}as\right)+a^2+1}$$

$$=\sqrt{\frac{5}{4}\left(s-\frac{4}{5}a\right)^2+\frac{a^2}{5}+1}$$

$$s=\frac{4}{5}a,\quad t^2=\frac{1}{4}\left(\frac{4}{5}a\right)^2+1$$

$$=\frac{1}{\cancel{4}}\cdot\frac{\cancel{16}^{\,4}}{25}a^2+1$$

$$=\frac{4}{25}a^2+1$$

$$t=\pm\sqrt{\frac{4}{25}a^2+1}$$

$$P\left(\frac{4}{5}a,\ \pm\sqrt{\frac{4}{25}a^2+1}\right)$$

$$AP_{min}=\sqrt{\frac{a^2}{5}+1}$$ //

C_1 $x^2-4y^2=-4$

$$\frac{x^2}{4}-y^2=-1$$

C_2 $x^2-4y^2=4$

$$\frac{x^2}{4}-y^2=1$$

$0<a<\frac{5}{2}$ のとき

$P(2,0)$

$A_{min}=|a-2|$

(2) 点 $P(s,t)$ とする

$$s^2-4t^2=4$$

$$AP=\sqrt{s^2-2as+a^2+\frac{s^2}{4}-1}$$

$$=\sqrt{\frac{5}{4}\left(s-\frac{4}{5}a\right)^2+\frac{a^2}{5}-1}$$

$$s=\frac{4}{5}a,\quad t^2=\frac{1}{\cancel{4}}\cdot\frac{\cancel{16}^{\,4}}{25}a^2-1$$

$$t^2=\frac{4}{25}a^2-1$$

$$t=\pm\sqrt{\frac{4}{25}a^2-1}$$

$$\frac{4}{25}a^2-1\geqq 0$$

$$a^2-\frac{25}{4}\geqq 0$$

$$\left(a+\frac{5}{2}\right)\left(a-\frac{5}{2}\right)\geqq 0$$

$a>0$ より, $a\geqq\frac{5}{2}$

$$P\left(\frac{4}{5}a,\ \pm\sqrt{\frac{4}{25}a^2-1}\right)$$

$$AP_{min}=\sqrt{\frac{a^2}{5}-1}\quad\left(a\geqq\frac{5}{2}\right)$$

(3) $0<a<\frac{5}{2}$

$$\sqrt{\frac{a^2}{5}+1}\geqq a-2$$

$$\frac{a^2}{5}+1\geqq a^2-4a+4$$

$$4a^2-20a+15\leqq 0$$

$$\frac{5-\sqrt{10}}{2}\leqq a\leqq\frac{5+\sqrt{10}}{2}$$

$0<a<\frac{5}{2}$ より

$$\frac{5-\sqrt{10}}{2}\leqq a\leqq\frac{5}{2}$$

$a\geqq\frac{5}{2}$ $a=\frac{5}{2}$

$\frac{5-\sqrt{10}}{2}\leqq a\leqq\frac{5}{2}$